Teaching Multivariable Calculus in High School
by Bob Enenstein, Carlmont High School, Belmont, CA
renenste@seq.org

This article describes how an ordinary public school managed to create enough demand to offer a multivariable calculus class. Because we are a California public school, we needed to find at least 25 qualified students. (Some schools may be able to find funds to offer this class to fewer students.) The main issues to be resolved are finding enough prepared students and developing a curriculum that allows them to complete BC Calculus by the end of their junior year.

The path starts with students taking Algebra I in seventh grade. I have been a vocal critic of this hyper-acceleration, which came about in California when the state decided that Algebra I would be the standard eighth grade class. The consequence was that most parents who thought that their child was above average wanted him or her to take Algebra I in seventh grade. Then those parents who thought that their child was average wanted him or her to take Algebra I in seventh grade for social reasons. The adverse result was that many students who could have done excellent work in Geometry through Calculus in high school were entering as ill-prepared Algebra II students. This was either because they took mathematics classes for which they were not ready or because they took watered-down classes, often both. Our school has stemmed the tide a little by articulating with the feeder schools and giving them a list of rigorous topics in Algebra I and Geometry that the students need to master before being recommended to take Algebra II in ninth grade. Conversely, we find one or two students a year that were under accelerated and we have them take two math classes as either a sophomore or a junior so that they can take multivariable calculus as a senior.

My personal feeling was that students who reach Multivariable Calculus should also be able to qualify for the American Invitational Mathematics Exam (AIME) exam. This means that a 9th grade Algebra II student should score 90 or higher on the AMC 10. Only a few of our students meet this requirement. However, we have a very active mathematics club and integrate discussions of mathematics contest problems into our curriculum. As a result, between 15 and 80 students per year over the last few years have qualified for the AIME. This is well above the number of students who would normally qualify from a school with only a couple of National Merit Semifinalists each year. (For more information on the AIME and other mathematics competitions, visit unl.edu/amc/index.shtml.) What we have found is that not all hyper-accelerated students continue on to BC Calculus as juniors. The ones that do have the study habits and desire to be successful in Multivariable Calculus can be even if they are not the most mathematically gifted.

What do you teach in Algebra II and Pre-Calculus to prepare students for BC calculus as juniors? In our Algebra II-Trigonometry class we review all the Algebra I topics—filling in large gaps for many students, cover all the Algebra II topics, and teach basic Trigonometry. The class has great breadth but little depth. We also offer a lower level
Algebra II class for students trying to complete the requirements for entry into state colleges.

The really interesting class is our Pre-Calculus class—Analytic Geometry–Calculus Honors—which has depth and conceptual work. We use Paul Foerster’s Pre-Calculus book and spend the first quarter doing statistics, probability, and data analysis. The students learn Excel and do mathematical modeling. The second quarter consists of vectors and a function approach to trigonometry. The third quarter curriculum is analytic geometry, polar coordinates, parametric equations, and infinite series. In the fourth quarter we do the first three chapters of Foerster’s Calculus book. Over the summer the students do sections from the old Protter-Morrey Calculus book, focusing on algebraic manipulations in calculus and a review of Algebra II and Pre-Calculus topics.

The final issue is administrative and probably different at every school. We set up a partnership with a local community college that offers Multivariable Calculus. They offer a section on our campus, and I am the teacher. It is open to the public, but we have not had an outside student yet. We are using the UC Berkeley Mathematics 53 syllabus, textbook, and assignments for the class, since many of our students apply for admission to UC Berkeley. With the Mathematics 53 syllabus and significant additional material, we more than satisfy all of the requirements of the community college. Concepts covered in this class include Parametric Equations and Polar Coordinates, Vectors and the Geometry of Space, Vector Functions, Partial Derivatives, Multiple Integrals, Vector Calculus, and Differential Equations. We teach the course over two semesters instead of one in order deeply explore concepts and give the students more time to absorb the material. This also allows a reduced homework load for students taking several other AP classes senior year.

In order to give students dual credit (high school and college) for Multivariable Calculus, we use an old approved course titled Independent Study Advanced Placement Calculus Honors. This has enabled us to avoid having to go through the time-consuming task of creating and getting approval for a new course at our school. We are trying to avoid this task because it is possible that the College Board may offer an AP Multivariable Calculus in the future.

I have set up a national database and helped Dan Teague form an interest group through the Mathematical Association of America (MAA) on the topic of teaching multivariable calculus in the high school. I encourage interested teachers to join me in forming a local network so we can support each other in this new endeavor. You can contact me at renenste@seq.org.